**Lift Forces in Bubbly Flows**


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## INTRODUCTION

Multiphase flows are found in a variety of engineering systems, two of the most notable categories being energy production and chemical/fossil fuel processing. Many energy production systems including jet engines, internal combustion engines, rockets [1], as well as the focus of this paper: light water reactors (LWRs) involve two-phase flows. In this type of fission reactor, light water is used as both coolant and neutron moderator, yielding a system whose safety and performance is bound to the spatiotemporal dynamics of the water and steam.

Broadly, there are two types of LWRs in use around the world: pressurized water reactors (PWRs) which in nominal operation are single phase systems, and boiling water reactors (BWRs) where boiling occurs inside the fuel channels of the reactor. Although little boiling occurs in PWRs under normal conditions, it is important to consider boiling and the subsequent two-phase flow for safety analyses and situations where the reactors experience DNB (Departure from Nucleate Boiling).

Engineered two-phase flow systems have complex geometry and often high Reynolds numbers which increase the computational cost of an analysis, and generally make direct numerical simulations of a flow field impossible even on the fastest computers today. Consequently, a number of models which seek to reduce this computational cost have been created over the years. One of these called the two-fluid model has become the standard in three dimensional CMFD (Computational Multiphase Fluid Dynamics) codes.

The two-fluid model splits each conserved quantity into two interspersed fields, one for each phase. Splitting one field into two requires additional interfacial closure relationships, as the boundaries are no longer explicitly resolved. In the two-fluid model, the momentum flux between the phases is governed by a number of these closure relationships. Generally, the net force on the dispersed phase is decomposed into inertial, added mass, buoyancy, drag, lift, and wall forces. In addition, there is a time-dependent Basset force, as well as turbulent dispersion effects. A diagram of a small bubble in shear flow is found in Fig. 1. The general form of force on a bubble can be written as [2]

$$\rho_b(\frac{4}{3}\pi a^3)\frac{du_b}{dt} = F_I + F_A + F_L + F_D + F_B \tag{1}$$

where $F_I$ is the inertial force, $F_A$ is the added mass term, $F_L$ is the lift force, $F_D$ is the drag force, and $F_B$ is the buoyancy force or in its full form

$$\begin{aligned}\frac{du_b}{dt} =& \frac{1+C_M}{\gamma + C_M}\left(\frac{\partial v}{\partial t} + (v\cdot\nabla)v\right) + \frac{C_L}{\gamma + C_M}(v-u_b)\times\omega \\ &+ \frac{1}{\gamma + C_M}\frac{3C_D}{8a}|v-u_b|(v-u_b) + \frac{\gamma - 1}{\gamma + C_M}g\end{aligned} \tag{2}$$

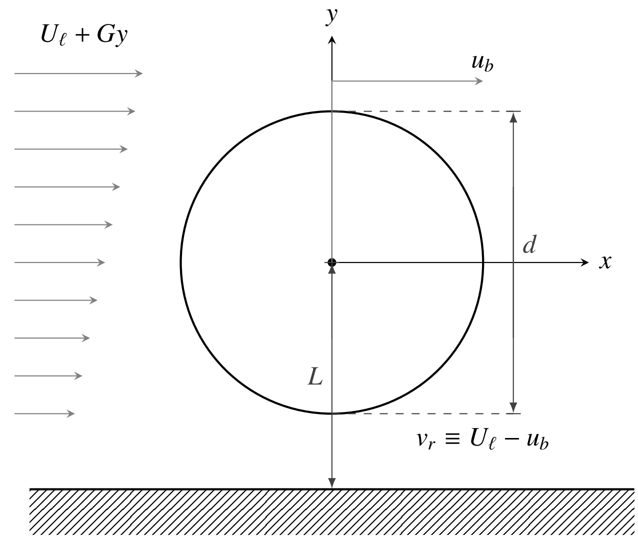


Fig. 1. Sketch of bubble in shear flow near wall with dimensional parameters

Here, $u_b$ is the bubble velocity, $v$ is the fluid velocity, $C_M$ is the added mass coefficient, $C_L$ is the lift coefficient, $C_D$ is the drag coefficient, $\gamma$ is the ratio of densities ($\rho_b/\rho$), $g$ denotes gravity, and $\omega$ stands for vorticity. In general, the coefficients are not constant although their functional dependence has not been explicitly stated. A few effects have been left out, including those due to time history, density gradients, and temperature gradients. Equation 2 is not based on a unified derivation of all these forces; rather, it is a composite of the various forces that have been found to act on bubbles. In the current study, we will use this superposition principle as an overall recast of the closure relations is not a trivial task.

One of the lesser understood momentum closures is that due to the transverse forces. These transverse forces are very critical for LWR applications as the radial motion of bubbles in the narrow channels between the fuel rods can have a large impact on the heat transfer and neutron moderation. A lift force found to act on dispersed particles in tube-flow was first quantitatively described by Segré and Silberberg [3]. In their 1962 paper, they find that a solid sphere in Poiseuille flow experiences radial forces, and that there is a stable equilibrium radial position at approximately $0.6r/R$ under certain flow conditions. The radial equilibrium position for particles in this experiment shows that there are competing forces in the transverse direction which are of similar magnitude. Their quantitative findings have been superseded by newer, more finely grained experimental data; however, they gave impetus to the following decades of theoretical and experimental work.

Transverse forces are perpendicular to the relative velocity of the bubble with respect to the fluid and are commonly decomposed into lift and wall force contributions in the CMFD community. The most common foundational lift model today is from Auton [4]; his model was derived under a number of assumptions, including: spherical shape, high particle $Re$, low $Sr$, and no external boundary. Using the base form of his model alone the aformentioned radial equilibrium positions can't be predicted, so a modified wall force term is introduced to counteract the lift. Auton's lift model is linear in $Sr$ and significantly overpredicts the shear lift force near a wall. In practice this creates stability issues for a CMFD code as both models are created to be similar in magnitude as a bubble approaches a boundary. A semi-empirical correction to Auton's model is developed below; it extends the applicability of his model by including dependency on $Re$ and distance from an external boundary.

## DEVELOPMENT OF NEW LIFT FORCE MODEL

Experimental and computational data from eight journal articles and technical reports has been compiled and converted into a uniform set of dimensionless parameters which can be found in Table I. This group of dimensionless numbers was chosen, as it occurs in theoretical work on the shear lift force, and most of the literature on adiabatic bubble lift force can be phrased in terms of them.[1] The deformation of a bubble and its orientation with respect to the primary flow will also strongly affect the lift force [5]; however, the work in this paper is restricted to roughly spherical bubbles.

TABLE I. Dimensionless parameters

| Name and definition | Description |
|---|---|
| $Re = \frac{dv_r}{\nu}$ | Particle reynolds number |
| $Sr = \frac{dG}{v_r}$ | Shear |
| $E = \frac{d}{2L}$ | Inverse dist. from wall |
| $C_L = \frac{F_L}{\rho \frac{\pi}{6} d^3 v_r G} = \frac{F_L}{\rho \frac{\pi}{6} \nu^2 Re^2 Sr}$ | Auton lift coeff. |
| $C_L^{Drag} = \frac{F_L}{\rho \frac{\pi}{8} d^2 v_r^2} = \frac{F_L}{\rho \frac{\pi}{8} \nu^2 Re^2}$ | Drag law type lift coeff. |

The lift force model for $C_L$ was built up in pieces. First, data far away from the wall ($E < 0.05$) was taken in order to construct model that was valid at high and low $Re$. A two-dimensional plot of the data at moderate $Re$ can be seen in Fig. 2. There is quite a bit of scatter in this plot as it contains points from a range of shear rates, and at varying distance from the wall. It also contains a line plot of the $C_L^{nowall}$ model, which is created to closely represent the lift force away from the wall (at $E = 0$). For comparison Legendre & Magnaudet's lift force model is also plotted. The difference in predictions between the two models is largest near a $Re$ of fifty, while both models approach $C_L \to 1/2$ for large Reynolds number.

After visualizing the data it was found that for low particle

[1] Including the effects of deformability requires at least one more parameter (e.g., Bond number)

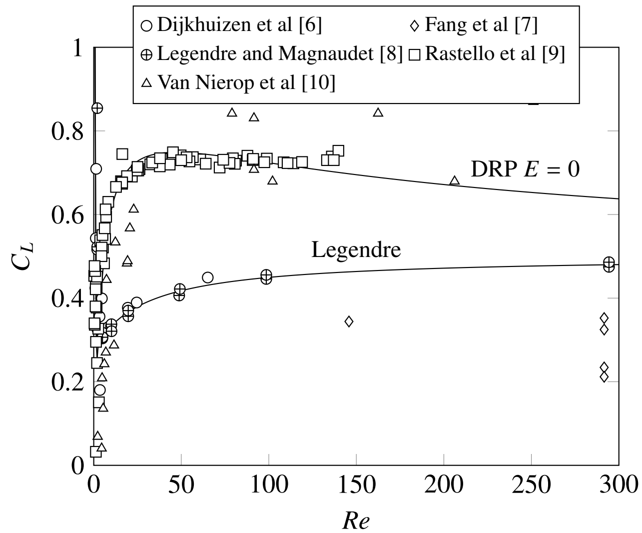


Fig. 2. Lift coefficient over $Re$ for data points $E < 0.05$, proposed model at $E = 0$, and Legendre & Magnaudet's model

$Re$ the model proposed by Legendre & Magnaudet [8] fit the data well. It is also based in theory and has correct asymptotic behavior for $Re \to 0$.

$$J(Sr, Re) = \frac{2.255}{(1 + 0.2Re/Sr)^{3/2}} \tag{3}$$

$$C_L^{low} = \frac{6J(Sr, Re)}{\pi^2 \sqrt{ReSr}} \tag{4}$$

At moderate to large $Re$ a new model given by Eqn. 5 was created. As $Re \to \infty$, $C_L \to 1/2$, so the model recovers the theoretical value of the lift coefficient. This model also encompasses another feature of the data: a local maximum near a $Re$ of 50. The high and low $Re$ models are combined in Eqn. 6 to produce a uniformly valid model away from the wall.

$$C_L^{high} = \frac{1}{2}\left(\frac{1 + 310/Re - 242/Re^2}{1 + 176/Re + 566/Re^2}\right) \tag{5}$$

$$C_L^{nowall} = \sqrt{C_L^{low\,2} + C_L^{high\,2}} \tag{6}$$

Finally, a model including the effect of distance from the wall was created by modifying $C_L^{nowall}$ with an additional term that decreases as a bubble approaches the wall. The functional form of this term was chosen so that as $E \to 0$, $C_L \to C_L^{nowall}$. This form was then fit against all of the data, and an exponent of $-2.3$ was found to minimize the squared error, so that the complete fit is given by Eqn. 7. Overall the model is found to fit the data well, with a mean squared error of 0.0075 for $Re > 5$.

$$C_L = C_L^{nowall} \log_2\left(\frac{E}{1 - E} + 2\right)^{-2.3} \tag{7}$$

The model is difficult to visualize in 2D, so Figure 3 shows a surface plot of the Daly, Ruggles, Pannala (DRP) model for $Re > 2$ and over all distances from wall ($0 \le E < 1$) along with a scatter plot of the data from the literature.

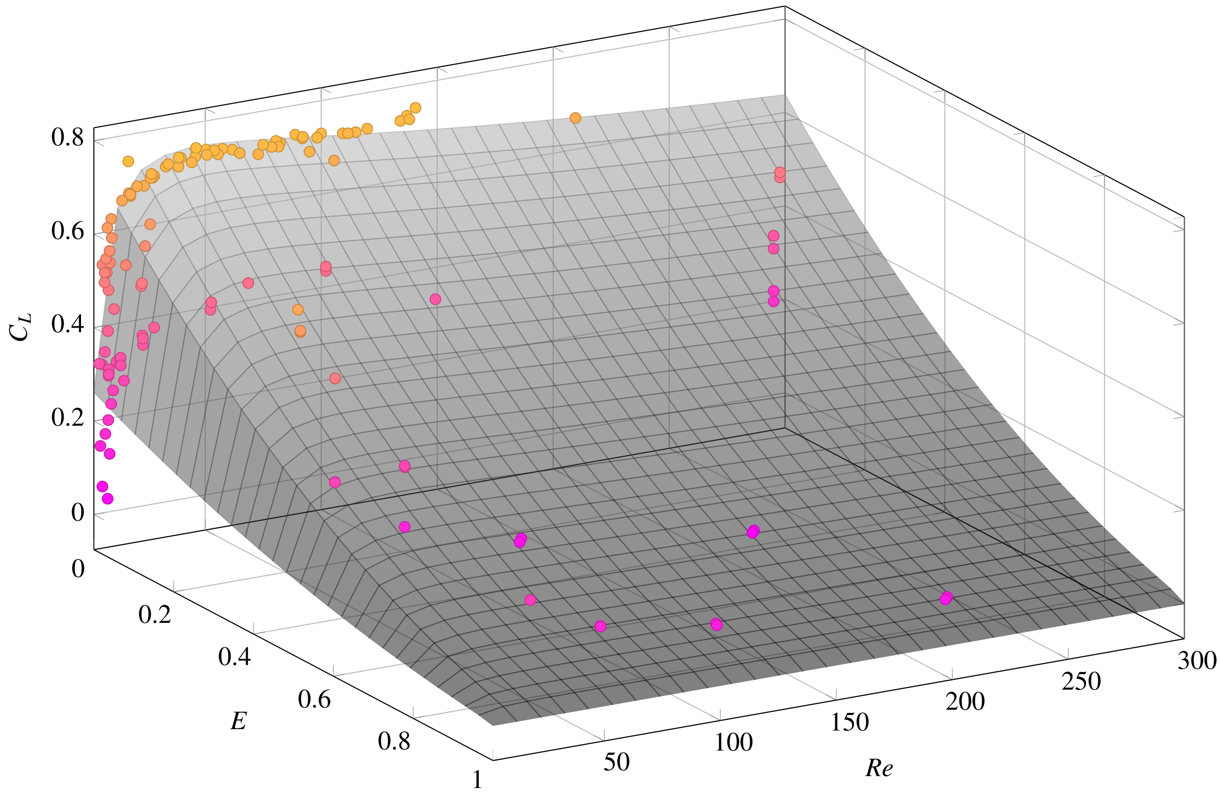


Fig. 3. 3D plot of DRP model and data points

**RESULTS**

In Figure 4 the lift force of the proposed model is applied in a turbulent channel, and compared with two other models: Auton's closure with constant $C_L$ as well as the model from Legendre and Magnaudet. In the standard closures, the lift force goes to a very large value as one goes to the wall and thus necessitates the wall force closure. The new closure ensures that the lift force decreases as a bubble approaches the wall, and thus it may work without the need for a wall force term or other work-around, such as an *ad hoc* cut-off of mesh close to the wall. The current closure has to be implemented in a CMFD code to test the validity and that is part of the future work.

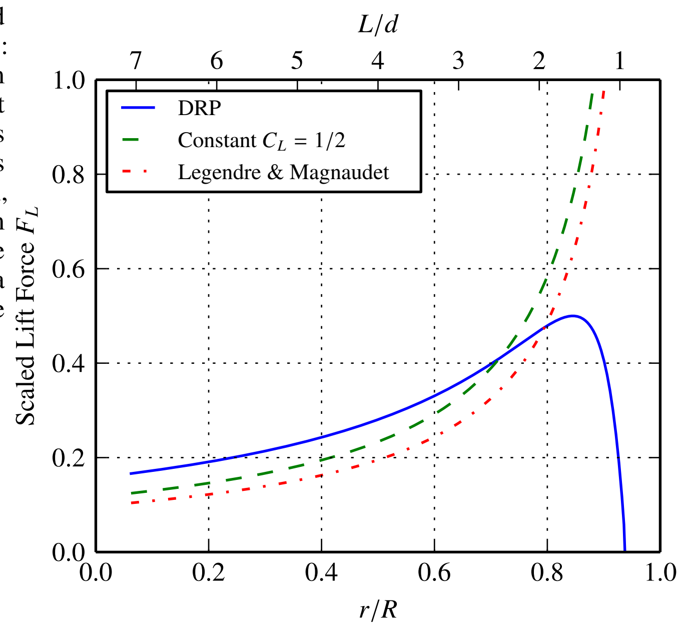


Fig. 4. Lift force comparison between different closures in a turbulent pipe flow

## NOMENCLATURE

- Upstream shear value: $G$
- Upstream uniform liquid velocity: $U_\ell$
- Liquid velocity far upstream of bubble: $v = U_\ell + Gy$
- Velocity of bubble: $u_b$
- Relative velocity: $v - u_b$
- Distance from wall to center of bubble: $L$
- Diameter of bubble: $d$
- Lift force: $F_L$

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