Estimation of the Shear-Induced Lift Force on a Single Bubble in Laminar and Turbulent Shear Flows Using Interface Tracking Approach

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November 20, 2014
Estimation of the Shear-Induced Lift Force on a Single Bubble in Laminar and Turbulent Shear Flows Using Interface Tracking Approach

Consortium for Advanced Simulation of Light Water Reactors
Undergraduate Research
presented by
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Under the direction of
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North Carolina State University
OUTLINE

- Introduction
- Problem Formulation
- Case Description
- V&V
- Results and Discussion
- Conclusions
- Acknowledgements
- References

Time: 327300

Bubble in high shear flow
INTRODUCTION

• Motivation
• Shear-induced lift force
Motivation

• Best estimate calculations of two-phase flow phenomena and heat transfer characteristics
• To understand on a fundamental level migration behavior of bubbles in coolant channels induced by interfacial forces
• Technological advances call for development of state-of-the-art models and simulation software
• Provide complementary drag and lift database for use in computational multiphase flow dynamics (CMFD) model development
• Ensure validation of simulations
Shear-induced Lift Force

- Lateral migration of bubbles
  - Turbulence production
  - Heat transfer
- Channel void fraction distribution
- Hibiki & Ishii (2007)
  - Extensive study summarizing the current status and improvements of lift force model development

Multi-bubble simulation (Bolotnov et al., 2011)
PROBLEM FORMULATION

- Lift force
- Governing equations
- Bubble control approach
Physics

- Lift force (Hibiki & Ishii 2007)
  - Relative velocity
  - Shear rate
  - Bubble rotational speed
  - Bubble surface boundary condition

Schematic of particle force due to relative motion in a non-uniform flow (a) viewed from an external frame of reference (b) viewed from the particle (Ervin & Tryggvason 1997)
Fundamental Forces

Classical drag equation expression

\[ F_D = \frac{1}{2} C_D \rho_l \nu_r^2 A \]

\[ F_B = (\rho_l - \rho_g) V_b g \]

\[ F_L = -C_L \rho_l V_b |\nu_r| \left| \frac{d\nu_l}{dy} \right| \]


• How to estimate???
• Improved functional form?
Bubble Control Algorithm

- Control Solution
  - PID-based controller
  - Control bubble’s location at (statistically) steady state
  - Control forces balance lift and drag forces

\[ F_D = -(F_B + F_{xc}) \quad F_L = -F_{yc} \]
Control Expression

\[ CF_i^{(n+1)} = p_1 CF_i^{(n)} + p_2 \left[ CF_i^{(n)} + p_3 dx_i^{(n)} + p_4 dx^2_i^{(n)} + p_6 v_i^{(n)} \right] \]
Data Analysis Method

• Compute instantaneous drag and lift coefficient
  • At each time step, need:
    • Applied control force
    • Average bubble velocity
    • Liquid velocity experience by bubble
  • Average (quasi) steady state portion of drag and lift coefficient
  • Window averaging
CASE DESCRIPTION

- Motivation
- Overview
- Case setup
Motivation

- Nuclear reactor coolant channel
  - Low relative velocity (0.033 m/s to 0.074 m/s)
    - Based on drag balance for bubble (0.1mm to 0.5mm) in up flow
  - High shear rates ($10^2$ s$^{-1}$ to $10^3$ s$^{-1}$)
    - High average channel velocity (4 m/s to 5 m/s) and hydraulic diameter (1.2 cm)
  - Turbulent flow
    - High Reynolds number ($10^5$)
    - Spacer grid and mixing vanes
Overview

• Low shear rate laminar flows (1.0 s\(^{-1}\) to 10.0 s\(^{-1}\))
  • Demonstrate control phenomenon
  • Initial assessment of controller performance

• Transition laminar flows (20.0 s\(^{-1}\) to 110.0 s\(^{-1}\))
  • Low to high shear rates
  • Assess and ensure control stability as shear rate increases

• High shear rate flows (236.0 s\(^{-1}\), 470 s\(^{-1}\))
  • Laminar – verify control is stable for highest shear rates
  • Turbulent – compare result to laminar counterpart

• Larger domain low shear flow
  • Assess domain dimension effects on lift and drag coefficient

• PWR fluid conditions
  • Low to high shear rates
  • Compare with air-water counterpart
**Case Setup**

- **Low Shear Laminar**
  - 5 mm bubble
  - 25 mm × 25 mm × 12.5 mm
- **All other cases**
  - 1 mm bubble
  - 5 mm × 5 mm × 2.5 mm
- **Boundary Conditions (same for all cases)**
  - Prescribed uniform shear velocity profile inflow/outflow (x-normal planes)
    - Turbulent case had prescribed turbulent shear velocity profile
  - Moving wall no-slip (y-normal planes)
  - Periodicity (z-normal planes)

---

<table>
<thead>
<tr>
<th></th>
<th>Water</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Density (kg/m³)</strong></td>
<td>996.5</td>
<td>1.161</td>
</tr>
<tr>
<td><strong>Dynamic Viscosity (Pa-s)</strong></td>
<td>8.5439·10⁻⁴</td>
<td>1.858·10⁻⁵</td>
</tr>
</tbody>
</table>
Verification and Validation

- Mesh Study
- Comparison
# Mesh Study

<table>
<thead>
<tr>
<th></th>
<th>Coarse</th>
<th>Fine</th>
<th>Finest</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elements Across Bubble</strong></td>
<td>16</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td><strong>Element Width (m)</strong></td>
<td>6.250E-05</td>
<td>5.000E-05</td>
<td>4.167E-05</td>
</tr>
<tr>
<td><strong>Interface Thickness (elements)</strong></td>
<td>0.96</td>
<td>1.2</td>
<td>1.44</td>
</tr>
<tr>
<td><strong>Interface Thickness (m)</strong></td>
<td>6.0E-05</td>
<td>6.0E-05</td>
<td>6.0E-05</td>
</tr>
<tr>
<td><strong>Total Number of Elements</strong></td>
<td>256,000</td>
<td>500,000</td>
<td>864,000</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0.463</td>
<td>0.437</td>
<td>0.519</td>
</tr>
<tr>
<td>$C_l$</td>
<td>0.494</td>
<td>0.514</td>
<td>0.535</td>
</tr>
</tbody>
</table>

**CASL-U-2015-0047-000**
Mesh Study

Shear 10 s\(^{-1}\), 20 elements across bubble

Shear 10 s\(^{-1}\), 24 elements across bubble

- Wake oscillation
- Turbulence or resonance?
Mesh Study

- Window Averaging
- Average over period of oscillation

<table>
<thead>
<tr>
<th>Window</th>
<th>Time range (s)</th>
<th>$C_d$</th>
<th>Time range (s)</th>
<th>$C_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Window 1</td>
<td>1.0 – 2.0</td>
<td>0.438</td>
<td>2.325 – 2.815</td>
<td>0.514</td>
</tr>
<tr>
<td>Window 2</td>
<td>2.0 – 3.0</td>
<td>0.436</td>
<td>2.815 – 3.31</td>
<td>0.514</td>
</tr>
<tr>
<td>Window 3</td>
<td>3.0 – 3.985</td>
<td>0.438</td>
<td>3.31 – 3.795</td>
<td>0.510</td>
</tr>
</tbody>
</table>
Validation

*Jun Fang and Jinyong Feng Reynolds number comparison study with Tomiyama et al. 1998
Results and Discussion

• Low Shear Laminar
• Transition Shear Laminar
• High Shear
  • Laminar
  • Turbulent
• Large Domain Low Shear Laminar
• PWR Fluid Conditions
## Low Shear Laminar

<table>
<thead>
<tr>
<th>Shear</th>
<th>( C_d )</th>
<th>( C_d^1 ) (Spherical bubble)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.3172</td>
<td>0.3292</td>
</tr>
<tr>
<td>2.0</td>
<td>0.1721</td>
<td>0.1646</td>
</tr>
<tr>
<td>5.0</td>
<td>0.1852</td>
<td>0.1646</td>
</tr>
<tr>
<td>10.0</td>
<td>0.2364</td>
<td>0.1646</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shear</th>
<th>( C_l )</th>
<th>( C_l^2 ) (Spherical bubble, high viscosity)</th>
<th>( C_l^3 ) (Spherical bubble, numerical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.3807</td>
<td>0.288</td>
<td>0.4628</td>
</tr>
<tr>
<td>2.0</td>
<td>0.4202</td>
<td>0.288</td>
<td>0.4797</td>
</tr>
<tr>
<td>5.0</td>
<td>0.4072</td>
<td>0.288</td>
<td>0.4797</td>
</tr>
<tr>
<td>10.0</td>
<td>0.3796</td>
<td>0.288</td>
<td>0.4797</td>
</tr>
</tbody>
</table>

1. The drag coefficient values predicted by (Tomiyama et al., 1998) based on Reynolds number for spherical bubble
2. The lift coefficient values predicted by (Tomiyama et al., 2002) based on Reynolds number for spherical bubble
3. A semi-empirical correlation based on numerical and theoretical solutions (Legendre & Magnaudet, 1998)
Low Shear Laminar

- Region of zero velocity
- Small eddy production

Timestep: 5000
Shear 1.0 s⁻¹

Timestep: 5000
Shear 10.0 s⁻¹
Transition Shear Laminar

\[
(\nabla \times \mathbf{v})_y = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}
\]

Shear 50 s\(^{-1}\)
0.1 m/s relative velocity
Transition Shear Laminar

Shear 110 s\(^{-1}\)
0.1 m/s relative velocity

Scale different to show flow field around bubble
Transition Shear Laminar

- Trends agree with Legendre & Magnaudet (1998) observation
- Correlations are independent of shear rate except Legendre & Magnaudet (1998)

\[ C_L = \sqrt{\left( \frac{6}{\pi^2} \left( \frac{2.255}{(ReSr)^{0.5}} \left(1 + 0.2\frac{Re}{Sr}\right)^{1.5} \right) \right)^2 + \left( \frac{11 + 16/Re}{21 + 29/Re} \right)^2} \]

\[ Sr = \frac{\omega d}{v_r} \]
High Shear Laminar

- Vorticity development region
- Bubble-vorticity interaction
- Bubble slightly deformed
- Wake interaction with boundary

Shear 236 s$^{-1}$
High Shear Laminar

Shear 470 s\(^{-1}\)

- Vorticity development region
- More activity
- Bubble-vorticity interaction
- Bubble deformed
## High Shear Laminar Shear

<table>
<thead>
<tr>
<th></th>
<th>236</th>
<th>470</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_d$</td>
<td>0.443667</td>
<td>0.546103</td>
</tr>
<tr>
<td>$C_d^4$ (Spherical bubble)</td>
<td>0.21092</td>
<td>0.121368</td>
</tr>
<tr>
<td>$C_d^5$ (Clean bubble)</td>
<td>0.482251</td>
<td>0.399242</td>
</tr>
<tr>
<td>$C_d^6$ (Distorted bubble)</td>
<td>0.390999</td>
<td>0.812996</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>236</th>
<th>470</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_l$</td>
<td>0.487571</td>
<td>0.621903</td>
</tr>
<tr>
<td>$C_l^7$ (Spherical, high $\mu$)</td>
<td>0.288</td>
<td>0.288</td>
</tr>
<tr>
<td>$C_l^8$ (Spherical, numerical)</td>
<td>0.474666</td>
<td>0.484688</td>
</tr>
</tbody>
</table>

4. The drag coefficient values predicted by (Tomiyama et al., 1998) based on Reynolds number for spherical bubble

5. An empirical drag coefficient correlation for a clean bubble (Ishii & Chawla, 1979)

6. An empirical drag coefficient correlation for a distorted bubble (Ishii & Chawla, 1979)

7. The lift coefficient values predicted by (Tomiyama et al., 2002) based on Reynolds number for spherical bubble

8. A semi-empirical correlation based on numerical and theoretical solutions (Legendre & Magnaudet, 1998)
High Shear Turbulent

- Need turbulent shear velocity profile to prescribe to inflow/outflow boundaries
- Single phase turbulent shear case
  - Generate turbulence with high shear
  - Record turbulence evolution
  - Feed into two-phase case as inflow/outflow boundary condition

\[ \text{Initial Shear Rate (s}^{-1}\text{)} \begin{array}{|c|c|c|} \hline \text{Re}_\tau & 580 & 1250 \ \hline \end{array} \]

\[ u_\tau \delta \geq 127.3 \]

\[ Re = \frac{u_\tau \delta}{\nu_l} \]

\[ Re = \frac{\rho v_{avg} d}{\mu} \]

\[ u_\tau = \sqrt{\nu_l \left( \frac{dU_l}{dy} \right)_{y=0}} \]

Lu & Tryggvason (2006)
High Shear Turbulent

- Boundary layer mesh
  - Resolve turbulence production region sufficiently
- Bulk resolution
  - 20 elements across bubble

<table>
<thead>
<tr>
<th>d (m)</th>
<th>y (m)</th>
<th>y+ (m)</th>
<th>y+ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.500</td>
<td>0.500</td>
<td>1.309E-05</td>
<td>1.309E-05</td>
</tr>
<tr>
<td>0.600</td>
<td>1.100</td>
<td>1.571E-05</td>
<td>2.880E-05</td>
</tr>
<tr>
<td>0.720</td>
<td>1.820</td>
<td>1.885E-05</td>
<td>4.764E-05</td>
</tr>
<tr>
<td>0.864</td>
<td>2.684</td>
<td>2.262E-05</td>
<td>7.026E-05</td>
</tr>
<tr>
<td>1.037</td>
<td>3.721</td>
<td>2.714E-05</td>
<td>9.740E-05</td>
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<tr>
<td>1.244</td>
<td>4.965</td>
<td>3.257E-05</td>
<td>1.300E-04</td>
</tr>
<tr>
<td>1.493</td>
<td>6.458</td>
<td>3.908E-05</td>
<td>1.691E-04</td>
</tr>
<tr>
<td>1.792</td>
<td>8.250</td>
<td>4.690E-05</td>
<td>2.160E-04</td>
</tr>
</tbody>
</table>

\[ y^+ = \frac{y u_{\tau}}{v} \]
High Shear Turbulent

- Window averaging shows fully developed turbulent flow
- Mean velocity profile
  - Development of turbulent boundary layers

\[
U_i(t) = \frac{1}{N_w} \sum_{j=1}^{N_w} u_{m}^i(t + t_j)
\]

\[
k(t) = \frac{1}{N_w} \sum_{j=1}^{N_w} \sum_{i=1}^{3} \frac{1}{2} (u_{m}^{i'}(t + t_j))^2
\]

\[Re_T = 130\]
High Shear Turbulent

- Window averaging shows converged solution
- Mean velocity profile
  - Development of turbulent boundary layers

\[ Re_T = 191 \]
High Shear Turbulent

rot_V_y
4000
2000
0
-2000
-4000

velocity X
0.5
0.4
0.2
0
-0.2
-0.4
-0.5

Time: 266250
Shear 236 s^-1
High Shear Turbulent

- Bubble deformation
  - Sharp interface
  - Numerical divergence
- Control bubble shape
  - Temporarily increase surface tension
- Bubble velocity matches liquid velocity
  - Must reject data

Instantaneous relative velocity turbulent shear 236 s⁻¹
High Shear Turbulent

Bubble position and velocity turbulent shear 236 s$^{-1}$
High Shear Turbulent

Bubble control forces turbulent shear $236 \text{ s}^{-1}$
High Shear Turbulent

Time: 289750
Shear 470 s⁻¹
High Shear Turbulent

- Very high shear
  - Extreme bubble deformation
- Control Bubble Shape
  - Permanently increase surface tension
- Bubble position controller
  - Remains stable
  - Relative velocity stable

Instantaneous relative velocity turbulent shear 470 s\(^{-1}\)
High Shear Turbulent

Bubble X Position

Bubble Y Position

Bubble Z Position

Bubble X Velocity

Bubble Y Velocity

Bubble Z Velocity

Bubble position and velocity turbulent shear 470 s$^{-1}$
High Shear Turbulent

Bubble control forces turbulent shear $470 \text{ s}^{-1}$
# High Shear Turbulent

<table>
<thead>
<tr>
<th>Shear</th>
<th>Turbulent 236</th>
<th>Turbulent 470</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_d$</td>
<td>1.1717(^9)</td>
<td>0.7633</td>
</tr>
<tr>
<td>$C_d^{10}$ (Clean bubble)</td>
<td>0.5032</td>
<td>0.4171</td>
</tr>
<tr>
<td>$C_d^{11}$ (Distorted bubble)</td>
<td>0.5266</td>
<td>0.3474</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shear</th>
<th>Turbulent 236</th>
<th>Turbulent 470</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_l$</td>
<td>0.9998(^9)</td>
<td>0.9041</td>
</tr>
<tr>
<td>$C_l^{12}$ (Spherical, numerical)</td>
<td>0.4719</td>
<td>0.4827</td>
</tr>
</tbody>
</table>

\(^9\) The value presented here was obtained by ignoring the data at time steps where the relative velocity attained a value of 0 m/s.

\(^{10}\) An empirical drag coefficient correlation for a clean bubble (Ishii & Chawla, 1979)

\(^{11}\) An empirical drag coefficient correlation for a distorted bubble (Ishii & Chawla, 1979)

\(^{12}\) A semi-empirical correlation based on numerical and theoretical solutions (Legendre & Magnaudet, 1998)
Large Domain Low Shear Laminar

<table>
<thead>
<tr>
<th>Domain Size</th>
<th>$C_d$</th>
<th>$C_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.4404</td>
<td>0.5178</td>
</tr>
<tr>
<td>Large</td>
<td>0.4345</td>
<td>0.5022</td>
</tr>
</tbody>
</table>

- Good agreement
- Flow
  - Laminar
  - High shear?
## PWR Fluid Conditions

<table>
<thead>
<tr>
<th></th>
<th>Density (kg/m³)</th>
<th>Viscosity (Pa-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Water</strong></td>
<td>996.5</td>
<td>8.5439e-4</td>
</tr>
<tr>
<td><strong>Air</strong></td>
<td>1.161</td>
<td>1.858e-5</td>
</tr>
<tr>
<td><strong>Sat. Liquid @155bar</strong></td>
<td>594.134</td>
<td>6.82964e-5</td>
</tr>
<tr>
<td><strong>Sat. Vapor @155bar</strong></td>
<td>102.071</td>
<td>2.31169e-5</td>
</tr>
</tbody>
</table>
PWR Fluid Conditions

Bubble Control Forces Shear 20 s⁻¹
PWR Fluid Conditions

Shear $100 \text{ s}^{-1}$, $Re_{bub} = 768.1$
PWR Fluid Conditions

Bubble Control Forces Shear 100 s\(^{-1}\)
### PWR Fluid Conditions

<table>
<thead>
<tr>
<th>Shear Rate (s&lt;sup&gt;-1&lt;/sup&gt;)</th>
<th>20</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_d )</td>
<td>0.2972</td>
<td>0.2449</td>
</tr>
<tr>
<td>( C_d^{13} ) (Clean bubble)</td>
<td>0.3173</td>
<td>0.3248</td>
</tr>
<tr>
<td>( C_d^{14} ) (Distorted bubble)</td>
<td>0.3393</td>
<td>0.5388</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shear Rate (s&lt;sup&gt;-1&lt;/sup&gt;)</th>
<th>20</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_l )</td>
<td>0.0131</td>
<td>0.5563</td>
</tr>
<tr>
<td>( C_l^{15} ) (Spherical, numerical)</td>
<td>0.4924</td>
<td>0.4918</td>
</tr>
</tbody>
</table>

---

13 An empirical drag coefficient correlation for a clean bubble (Ishii & Chawla, 1979)
14 An empirical drag coefficient correlation for a distorted bubble (Ishii & Chawla, 1979)
15 A semi-empirical correlation based on numerical and theoretical solutions (Legendre & Magnaudet, 1998)
Conclusions

• Current lift and drag correlations
• Reliable method for numerically estimating lift and drag
• Low shear laminar validates method (5 cases)
• Transition shear flows (8 cases)
  • Qualitative understanding of flow field
  • Vorticity development
  • Bubble-vorticity interaction
• High shear flows (4 cases)
  • Laminar is not so laminar
  • Turbulent coefficients larger than laminar counterparts
    • Numerical instabilities at high shears
• PWR-conditions (2 cases)
  • Low-medium shear rates studied
  • Bubble-wake interaction important
Conclusions

• Overall conclusions
  • Insight into bubble control method for estimating lift and drag
  • Path forward to simulating more PWR-like conditions
    • High shear
    • Low relative velocity
    • High Reynolds number turbulent
    • Saturated fluid properties
  • Advance in computing capacity can increase efficiency and throughput
    • Direct numerical simulations & Interface tracking methods
    • High performance computing
  • Nuclear reactor Thermal Hydraulics
    • State-of-the-art models and correlations
    • Accurate predictions of 3D two-phase flow systems for transient analysis
Future Work

• Relative velocity definition (liquid velocity value)
• Complex interaction between wake and shear field
• Domain size for very high shear case
• PWR conditions at higher shear rates, larger domains
• Further mesh resolution study
• Bubble deformation and projected area definition
• Parallelization of time-varying-type boundary condition
• Turbulent shear 236 s\(^{-1}\) using same ST as 470 s\(^{-1}\)
• F\(_L\) & F\(_D\) function form development
Acknowledgements

- DOE Energy Innovation Hub, Consortium for advanced simulation of light water reactors (CASL)
- National Science Foundation
- U.S. Nuclear Regulatory Commission (NRC)-funded fellowship
- U.S. NRC-faculty development program
  - Computational resources extensively used in this research
- Altair Engineering Inc.
  - Acusim linear algebra solution library
- Simmetrix Inc.
  - Meshing and geometric modeling libraries
- Mr. Jun Fang and Jinyong Feng
References (1)


References (2)

Appendix
Literature Review

• Analytical
  • Saffman 1965; 1968

• Numerical
  • Tomiyama et al. 1993
  • Takagi & Matsumoto 1995
  • Ervin & Tryggvason 1997
  • Legendre & Magnaudet 1998
  • Zhongchun et al. 2014

• Experimental
  • Segre & Silberberg 1962a; 1962b
  • Kariyasaki 1987
  • Tomiyama et al. 2002

Tomiyama et al. 2002

Ervin & Tryggvason 1997
Literature Review

- Turbulent
  - Wang & Maxey 1993
  - Sene et al. 1994
  - Sridhar & Katz 1999
  - Mazzitelli & Lohse 2003
  - Giusti et al. 2005
  - Wang et al. 1987

- Main Ideas
  - Bubble size
  - Deformability
  - Lift force direction
  - Bubble’s wake
  - Vortex entrainment

\[ Re_b = \frac{\rho_l v_r d_b}{\mu_l} \]

Tomiyama et al. 2002

\[ Eo = \frac{g(\rho_l - \rho_g)d_b^2}{\sigma} \]

Fig. 6. \( C_T \) for small bubbles.

Fig. 7. \( C_T \) for large bubbles.
## High Shear Turbulent

### Minimum $v_{rel}$ limit (m/s)

<table>
<thead>
<tr>
<th>Minimum $v_{rel}$ limit (m/s)</th>
<th>$C_d$</th>
<th>$C_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>1.2205</td>
<td>1.0856</td>
</tr>
<tr>
<td>0.10</td>
<td>1.1717</td>
<td>0.9998</td>
</tr>
<tr>
<td>0.12</td>
<td>1.0966</td>
<td>0.9115</td>
</tr>
</tbody>
</table>

### Time Range (s)

<table>
<thead>
<tr>
<th>Time Range (s)</th>
<th>$C_d$</th>
<th>$C_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010 – 0.160</td>
<td>1.2999</td>
<td>0.8106</td>
</tr>
<tr>
<td>0.290 – 0.395</td>
<td>0.9703</td>
<td>0.8032</td>
</tr>
<tr>
<td>0.410 – 0.509</td>
<td>0.8162</td>
<td>0.9923</td>
</tr>
<tr>
<td>0.568 – 0.650</td>
<td>1.4903</td>
<td>1.0316</td>
</tr>
<tr>
<td>Average of all Time Ranges</td>
<td>1.1442</td>
<td>0.9094</td>
</tr>
</tbody>
</table>
Governing Equations

- Continuity $u_{i,j} = 0$
- Momentum $\rho u_{i,t} + \rho u_j u_{i,j} = -p_i + \tau_{ij,j} + f_i$
- Viscous Stress Tensor $\tau_{ij} = 2\mu S_{ij} = \mu(u_{i,j} + u_{j,i})$
- Re-distancing Procedure (Sussman & Fatemi, 1999)

$$\frac{\partial d}{\partial t} = S(\phi) \left[ 1 - \nabla d \right]$$
$$\frac{\partial d}{\partial t} + w \cdot \nabla d = S(\phi)$$
$$w = S(\phi) \frac{\nabla d}{d - 1}$$

$$S(\phi) = \begin{cases} 
-1, & \phi < -\varepsilon_d \\
\frac{\phi}{\varepsilon_d} + \frac{1}{\pi} \sin \left( \frac{\pi \phi}{\varepsilon_d} \right), & |\phi| < \varepsilon_d \\
1, & \phi > \varepsilon_d
\end{cases}$$
Transition Shear Laminar

Shear 110 s\(^{-1}\)
0.1 m/s relative velocity

Shear 50 s\(^{-1}\)
0.1 m/s relative velocity
NUMERICAL METHOD

- Navier-Stokes Solver
- Solution Visualization
**PHASTA**

  - Spatial and temporal discretization of N-S equations

- Brackbill et al. (1992)
  - Continuum Surface Tension Model

  - Level set method for interface tracking

- Brackbill et al. (1992)
  - Continuum Surface Tension Model

### Equations

\[
F_{sa}(x_s) = \sigma \kappa(x_s)\hat{n}(x_s)
\]

\[
\kappa = (\nabla \cdot \hat{n})
\]

\[
\hat{n} = \frac{\nabla \varphi}{|\nabla \varphi|}
\]

**Parallel**

**Hierarchic, higher-order accurate**

**Adaptive**

**Stabilized, finite element method**

**Transient**

**Analysis flow solver**

- Sussman et al. (1998), Sethian (1999)
  - Level set method for interface tracking
Level Set Method

- Solving geometric equation
- Gas-liquid interface is zero level set of smooth function, $\varphi$
- Finite thickness interface
  - Smoothed Heaviside kernel function for property definition
- Re-distancing Procedure (Sussman & Fatemi, 1999)

$$D\varphi = \partial\varphi + u \cdot \nabla \varphi = 0$$

$$H_\varepsilon(\varphi) = \begin{cases} 
0 & , \varphi < -\varepsilon \\
\frac{1}{2} \left[ 1 + \frac{\varphi}{\varepsilon} + \frac{1}{\pi} \sin \left( \frac{\pi \varphi}{\varepsilon} \right) \right] & , |\varphi| < \varepsilon \\
1 & , \varphi > \varepsilon 
\end{cases}$$

$$\xi(\varphi) = \xi_1 H_\varepsilon(\varphi) + \xi_2 \left( 1 - H_\varepsilon(\varphi) \right)$$
Visualization

• Kitware’s ParaView
  • Open-source
  • Multi-platform
  • Batch processing
  • Large datasets
  • Distributed memory

ParaView visualization of flow in/around F-35 vertical landing
Acquired from Paraview.org
Bubble Control Application

• “Whole Domain” Method
  • Alter gravity throughout whole domain

\[ F_D = - (F_B + F_{xc}) = - \left( (\rho_l - \rho_g) V_b (g + g_{xc}) \right) = \frac{1}{2} C_D \rho_l v_r^2 A \]

\[ F_L = -F_{yc} = - (\rho_l - \rho_g) V_b g_{yc} = -C_L \rho_l V_b \left| v_r \right| \left| \frac{dv_l}{dy} \right| \]

• “Bubble” Method
  • Apply volumetric force only to bubble interior

\[ F_D = -F_{xc} = \frac{1}{2} C_D \rho_l v_r^2 A \]

\[ F_L = -F_{yc} = -C_L \rho_l V_b \left| v_r \right| \left| \frac{dv_l}{dy} \right| \]
PID-based Control

- **Proportional**
  - Bubble location with respect to initial location

- **Integral**
  - Historical average of control force

- **Derivative**
  - Bubble velocity and acceleration

Ziegler & Nichols (1942)
## Computational Cost

<table>
<thead>
<tr>
<th>Case</th>
<th>Shear 50</th>
<th>Shear 100</th>
<th>Shear 236 (L)</th>
<th>Shear 236 (T)</th>
<th>Number of Cores</th>
<th>Total CPU-hours for a Single Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Simulation Time (s)</td>
<td>1.3385</td>
<td>3.478</td>
<td>1.541</td>
<td>0.6915</td>
<td>128</td>
<td>384</td>
</tr>
<tr>
<td>Total Number of Time steps</td>
<td>30,750</td>
<td>120,000</td>
<td>120,000</td>
<td>116,950</td>
<td>128</td>
<td>384</td>
</tr>
<tr>
<td>Total Wall Clock Time (d)</td>
<td>1.09</td>
<td>4.85</td>
<td>5.42</td>
<td>14.91</td>
<td>128</td>
<td>384</td>
</tr>
<tr>
<td>Number of Cores</td>
<td>128</td>
<td>128</td>
<td>128</td>
<td>64</td>
<td>128</td>
<td>384</td>
</tr>
<tr>
<td>Total Number of Mesh Elements</td>
<td>500,000</td>
<td>500,000</td>
<td>500,000</td>
<td>535,000</td>
<td>128</td>
<td>384</td>
</tr>
<tr>
<td>CPU-Hours Estimate</td>
<td>3,334.8</td>
<td>14,914.5</td>
<td>16,636.4</td>
<td>22,899.2</td>
<td>128</td>
<td>384</td>
</tr>
</tbody>
</table>

- **AMD Opteron 6276**
  - 16 Core
  - 2.3 GHz